
Jet and Rocket Propulsion

AE4451

LECTURE 40: Pre-final review 1

Overview

- what we saw in Lecture 39
 - turbines and turbine analysis
 - reaction: difference between $R = 0, 50\%$
 - multistage considerations, geometry
 - stresses, operational limits
 - conductive cooling

- today
 - exam details
 - revision
 1. LREs: using heat transfer coefficients and correlations
 2. sizing considerations: using the mass flow parameter
 3. graphical interpretation

★ announcement ★

This Friday's lecture will be by TA Ahmed Wahid; discussion of vortical structures in canonical flows like jets, swirling jets, wakes, jets with cross flow

Exam details

- 20% of overall grade
- Date and time: **April 28th, 2:40 – 5:30 pm**
Location: TBA
- compared to midterm:
 - material and concepts from the second half of semester will be tested (~70%)
 - open book
 - equation sheet will be provided latest by Monday April 24, however:
 - not every single equation on it; annotate if you need to
 - print out or refer to the pdf during the exam
 - highly advised to make your own version, with as much info as you need
 - evaluate expressions fully (unless told to leave expressions in terms of a particular parameter)
 - more partial credit awarded if correct reasoning but incorrect values, however:
 - please show your working/reasoning, to make grading easier
 - at some points, you may be provided with values you can use for your evaluations

Liquid rocket engines

Calculations using heat transfer coefficients and correlations

- previously, we encountered semi-empirical correlations for liquid rocket engines

Lec. 26

$$\text{Nu}_D \propto \text{Re}^{0.8} \text{Pr}^{0.4}$$

e.g. hot gas side
Bartz correlation (1965)

$$\text{Nu}_{D_H} \propto \text{Re}^{0.8} \text{Pr}^n$$

e.g. coolant side

$$\text{Nu} = 0.332 \text{Re}^{1/2} \text{Pr}^{1/3}$$

flat plate, laminar
(subsonic) flow

- let's see how we can apply this to the following problem:

Consider a liquid propellant engine combustion chamber, diameter 0.45 m

- cooled by regenerative cooling to maintain an outer wall temperature of 300 K
- temperature and pressure of combustion chamber maintained at 3000 K and 0.75 MPa
- heat loss due to gas radiation = 25% of total heat loss
- wall thickness = 5.2 mm; wall thermal conductivity = 21 W/m · K,

Q. determine the inner wall temperature at steady-state

use: $\text{Re} = 10^6$, $\text{Pr} = 0.73$, gas thermal conductivity $k_g = 0.17 \text{ W/m} \cdot \text{K}$

Liquid rocket engines

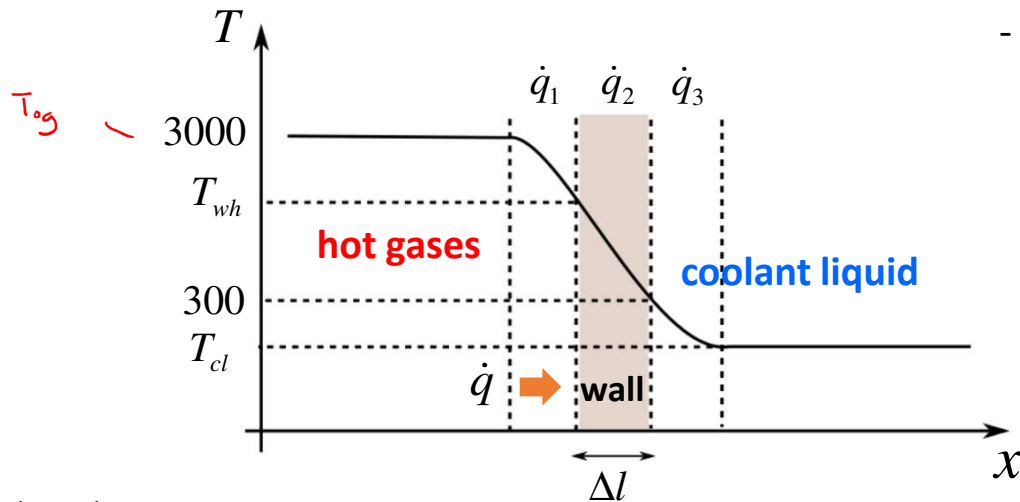
Calculations using heat transfer coefficients and correlations

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- recall heat flux expression

$$\dot{q} = \dot{q}_{convective} + \dot{q}_{radiative}$$

$$\dot{q} = h_g (T_{aw} - T_{wh}) + \dot{q}_{radiative}$$

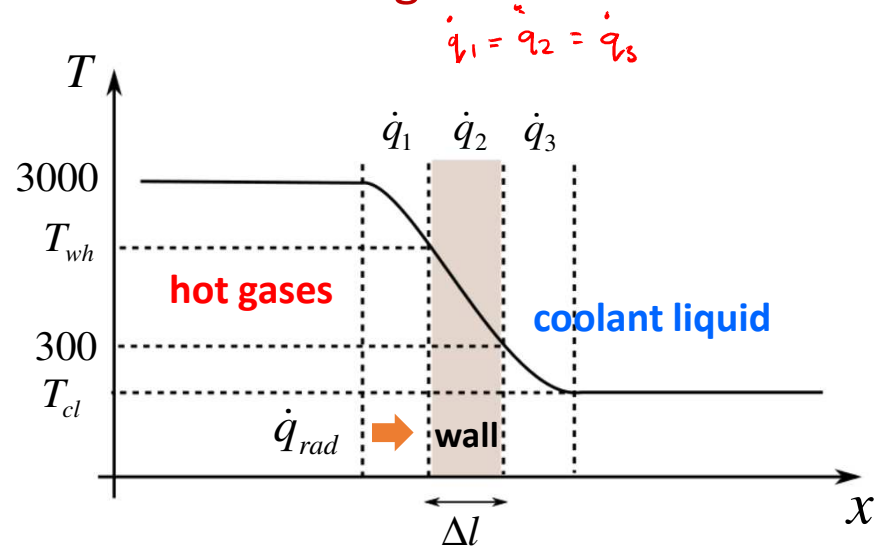
- recall expression for adiabatic wall temperature

adiab. wall temp. $T_{aw} = rT_{og} + T_{\infty}(1-r)$

- assume here $r = 1$, such that $T_{aw} = T_{og}$

Liquid rocket engines

Calculations using heat transfer coefficients and correlations



- overall heat flux expression to use becomes

$$\dot{q} = \underbrace{h_g (T_{og} - T_{wh})}_{\text{convective}} + \dot{q}_{radiative}$$

- also given: $\dot{q}_{radiative} = 0.25\dot{q}$

$$\Rightarrow \dot{q} = \frac{4}{3} h_g (T_{og} - T_{wh})$$

steady flux

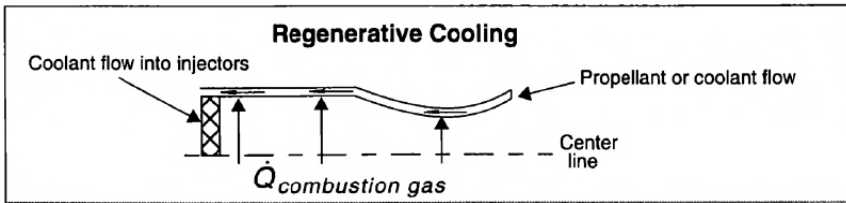
- we have another expression for the heat flux, from conductivity across the wall, i.e.

$$\dot{q} = \frac{-k}{\Delta l} (T_{wc} - T_{wh})$$

Lec. 24

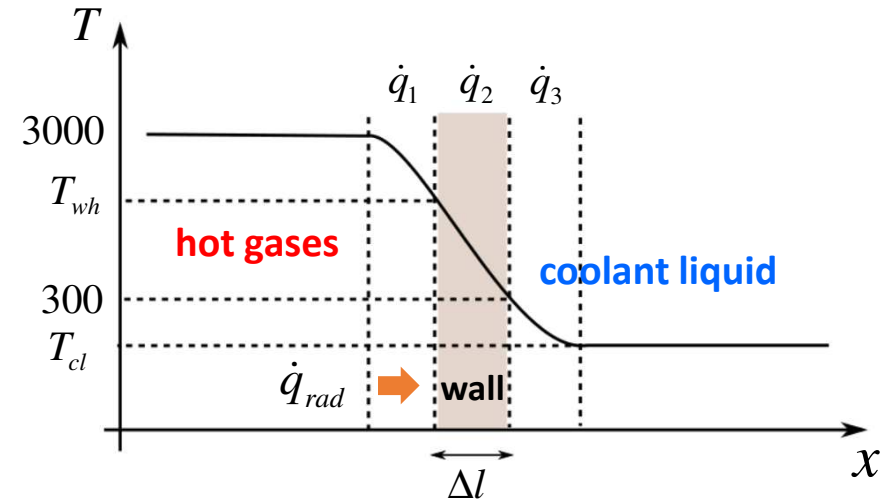
- notice here we don't have some info on the coolant...but not needed
- equate fluxes

$$\Rightarrow \frac{4}{3} h_g (T_{og} - T_{wh}) = \frac{-k}{\Delta l} (T_{wc} - T_{wh})$$



Liquid rocket engines

Calculations using heat transfer coefficients and correlations



- expression for hot wall temperature

$$T_{wc} - T_{wh} = - \underbrace{\frac{4}{3} h_g \frac{\Delta l}{k}}_a (T_{og} - T_{wh})$$

let's temporarily call this "a"

$$T_{wh} = \frac{a T_{og} - T_{wc}}{a - 1}$$

- notice that we were given not a h_g (heat transfer coefficient) but thermal conductivity k_g

- we have correlation expressions, e.g. for the coolant:

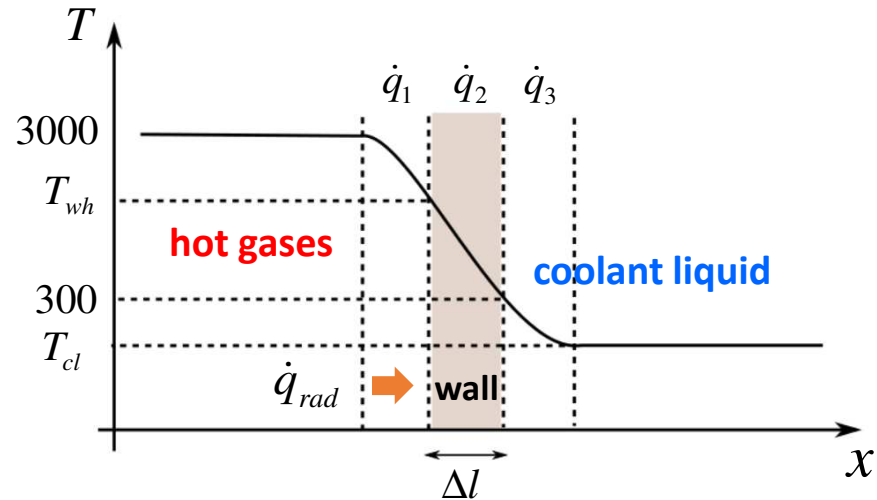
Lec. 26

$$h_{cl}(x) = C \frac{k_{cl}}{D} \left(\frac{\rho_{cl} u_{cl} D}{\mu_{cl}} \right)^{0.8} \left(\frac{\mu_{cl} c_{p,cl}}{k_{cl}} \right)^n$$

$\begin{cases} C = 0.018, n = 0.37 \\ C = 0.023, n = 0.33 \end{cases}$

Liquid rocket engines

Calculations using heat transfer coefficients and correlations



- for the gas side, we will use

$$Nu \equiv \frac{hL}{k}$$

convective
conductive

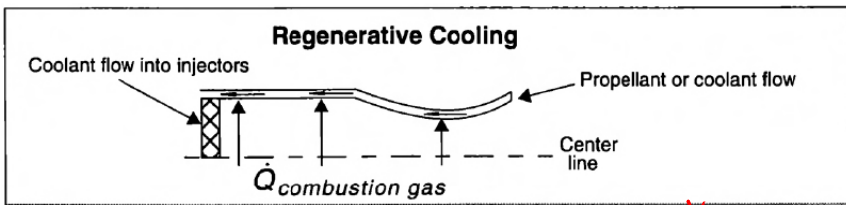
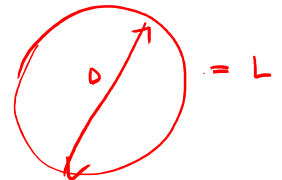
correlation form: $Nu = b Re^m Pr^n$ with given constants b, m, n

- for $b = 0.023, m = 0.8, n = 0.33$, *given values to use*

$$Nu = 0.023 \times (10^6)^{0.8} \times (0.73)^{0.33}$$

$$Nu = 1308$$

$$\Rightarrow h_g = Nu \frac{k_g}{L} = 1308 \times \frac{0.17}{0.45} = 494.15 \text{ W/m}^2$$

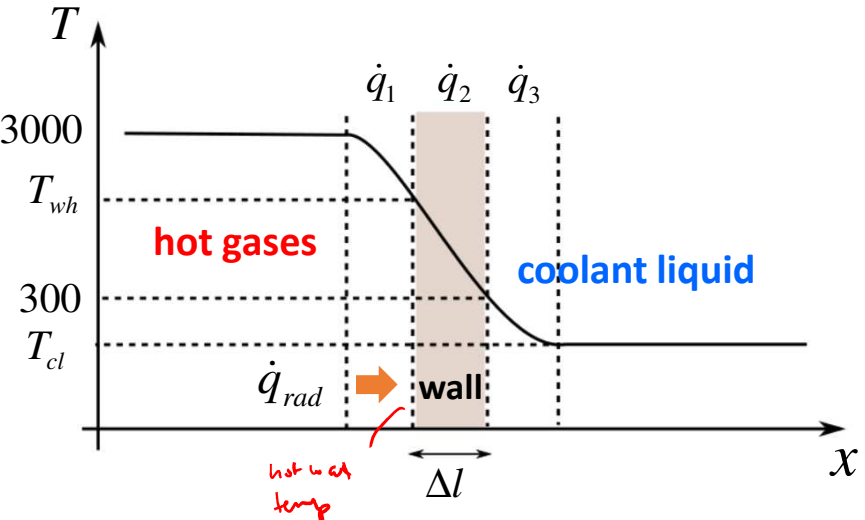


- notice that the relevant dimension L was the chamber diameter

Q. what does the value of Nu tell us about the flow?

Liquid rocket engines

Calculations using heat transfer coefficients and correlations



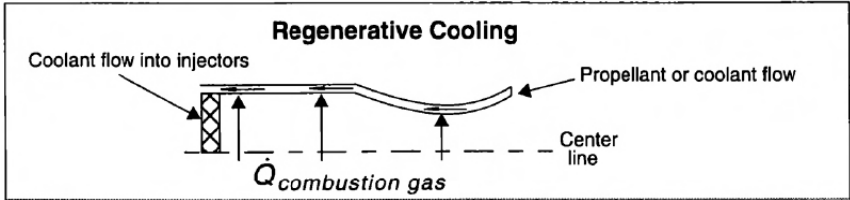
- from our original expression

$$T_{wc} - T_{wh} = -\frac{4}{3} h_g \frac{\Delta l}{k} (T_{og} - T_{wh})$$

$$a = -\frac{4}{3} h_g \frac{\Delta l}{k} = -\frac{4}{3} \times 494.15 \times \frac{5.2 \times 10^{-3}}{21} \text{ m}$$

w/m².k

$$a = -0.163$$



$$\Rightarrow T_{wh} = \frac{aT_{og} - T_{wc}}{a-1} = \frac{-0.163 \times 3000 - 300}{-0.163 - 1}$$

300K

$$T_{wh} = 678.4 \text{ K}$$

Sizing considerations

Interpretations using the mass flow parameter

MFP maximum when M = 1

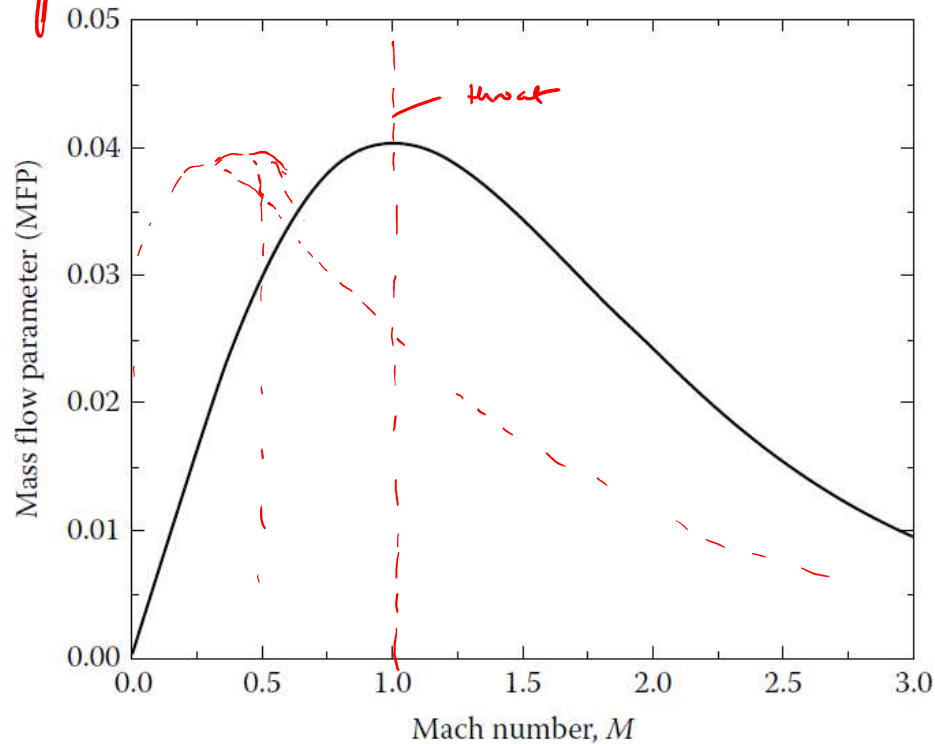
- previously, we encountered the mass flow parameter, MFP

$$MFP = \frac{\dot{m}}{A} \frac{\sqrt{T_o}}{p_o} = M \sqrt{\frac{\gamma}{R} \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

Lec. 29

- suppose we have the following plot of the MFP vs M for air

Q. what happens to the mass flow if we impose the constraint that the flow at the smallest section can only be at M = 0.5?



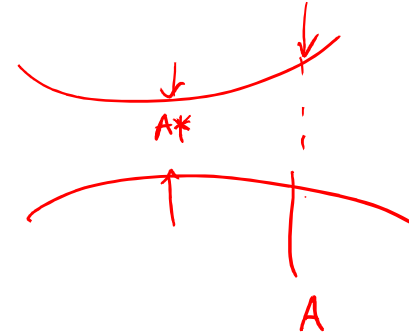
Sizing considerations

Interpretations using the area-Mach number relation

- we also previously encountered the following relation

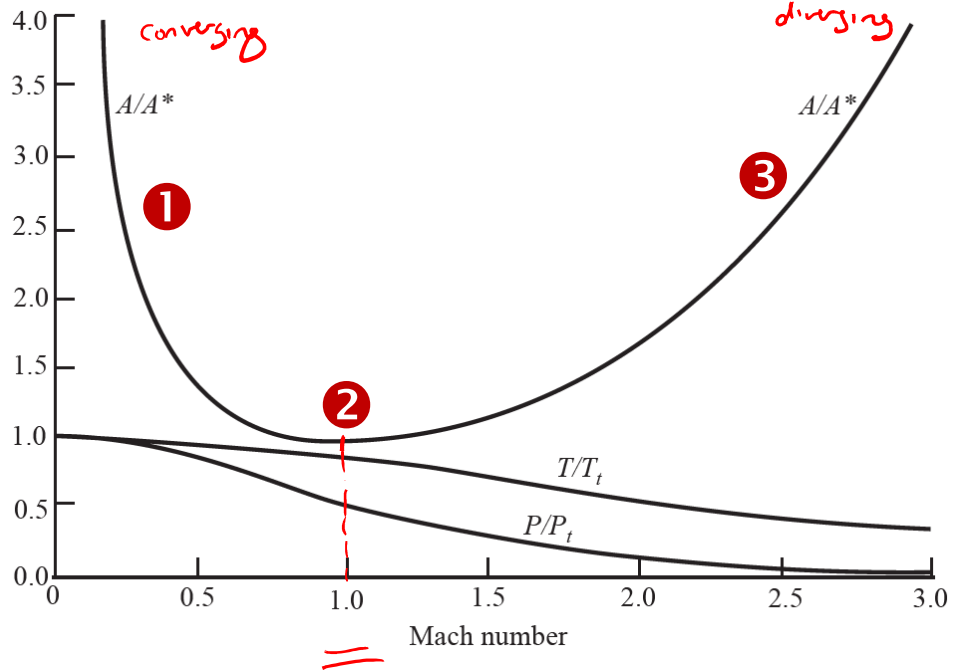
$$\frac{A}{A^*} = \frac{1}{M} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Lec. 30



- suppose we had to describe the duct based on the change of the area ratio and Mach number

Q. how would you label the duct at the positions indicated? converging, diverging...?



es. Mathngly

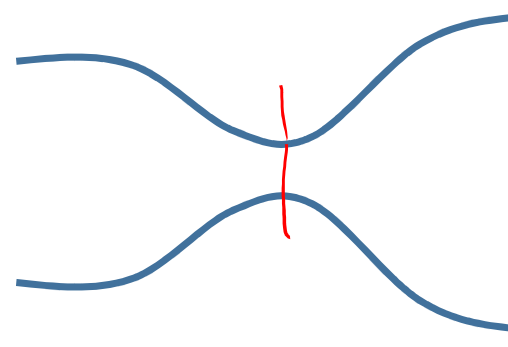
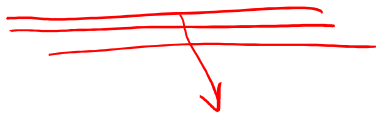
Sizing considerations

Calculations using the area-Mach number relation

- let's now consider the following situation:
 - we are asked to design a converging-diverging nozzle such that:
 - the exit Mach number is **3.5**
 - combustion products with $\gamma = 1.25$ are entering at pressure **12 MPa** and temperature **2500 K**
 - an exit diameter of the nozzle of **1.45 m**

1. what is the ratio of throat area to the exit area?
2. what is the maximum mass flow rate?

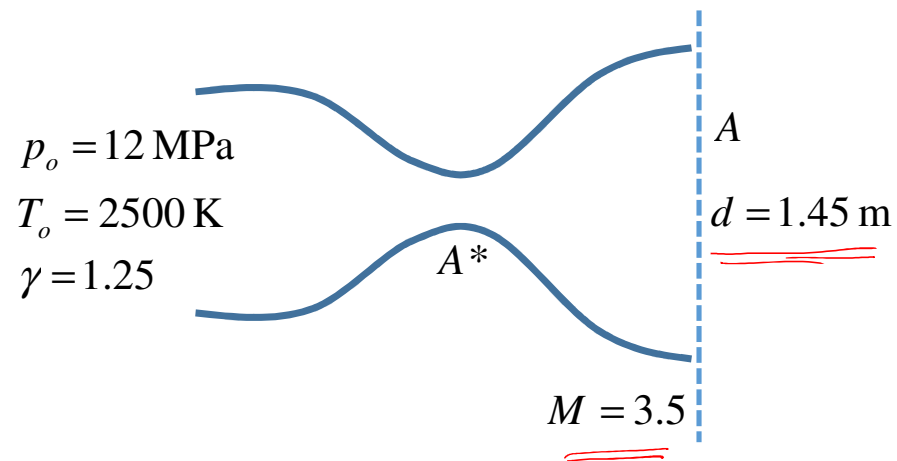
- use $R = 287 \text{ J}/(\text{kg}\cdot\text{K})$, value for dry air



Sizing considerations

Calculations using the area-Mach number relation

1. what is the ratio of throat area to the exit area?



- this relation allows us to find that area relationship at any point in the duct if we know M

$$\frac{A}{A^*} = \frac{1}{M} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

- so at exit, pretty straightforward

$$\frac{A}{A^*} = \frac{1}{3.5} \left[\frac{2}{1.25+1} \left(1 + \frac{1.25-1}{2} 3.5^2 \right) \right]^{\frac{1.25+1}{2(1.25-1)}}$$

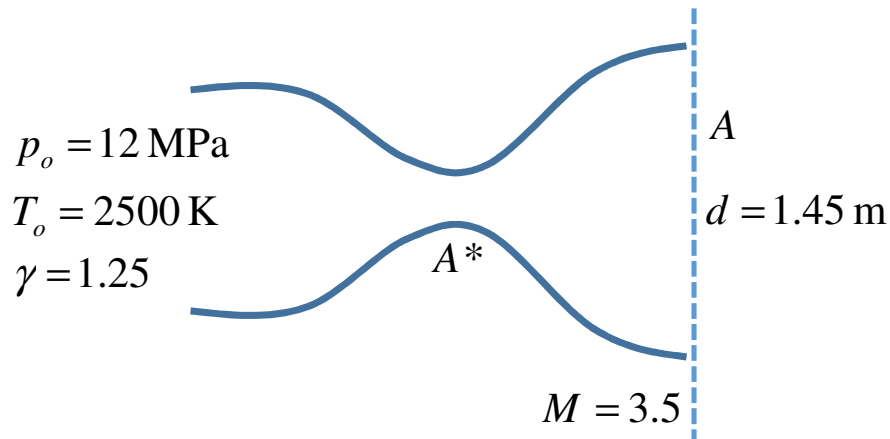
$$\frac{A}{A^*} = 10.98$$

$$A = 1.65 \text{ m}^2, A^* = 0.15 \text{ m}^2$$

Sizing considerations

Calculations using the area-Mach number relation

2. what is the maximum mass flow rate?



- this is where the MFP expression comes in handy:

$$MFP = \frac{\dot{m}}{A} \frac{\sqrt{T_o}}{p_o} = M \sqrt{\frac{\gamma}{R}} \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

- maximum mass flow rate for $M = 1$ at throat

$$\frac{\dot{m}}{A^*} \frac{\sqrt{T_o}}{p_o} = \sqrt{\frac{\gamma}{R}} \left(1 + \frac{\gamma-1}{2} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

...but if calculating a corrected mass flow rate, will reduce this M to leave a margin and avoid choking

$$\Rightarrow \dot{m} = \sqrt{\frac{\gamma}{RT_o}} \left(1 + \frac{\gamma-1}{2} \right)^{\frac{\gamma+1}{2(\gamma-1)}} p_o A^*$$

$$\Rightarrow \dot{m} = \sqrt{\frac{1.25}{287 \times 2500}} \left(1 + \frac{1.25-1}{2} \right)^{\frac{1.25+1}{2(1.25-1)}} \underline{12 \times 10^6} \times 0.15$$

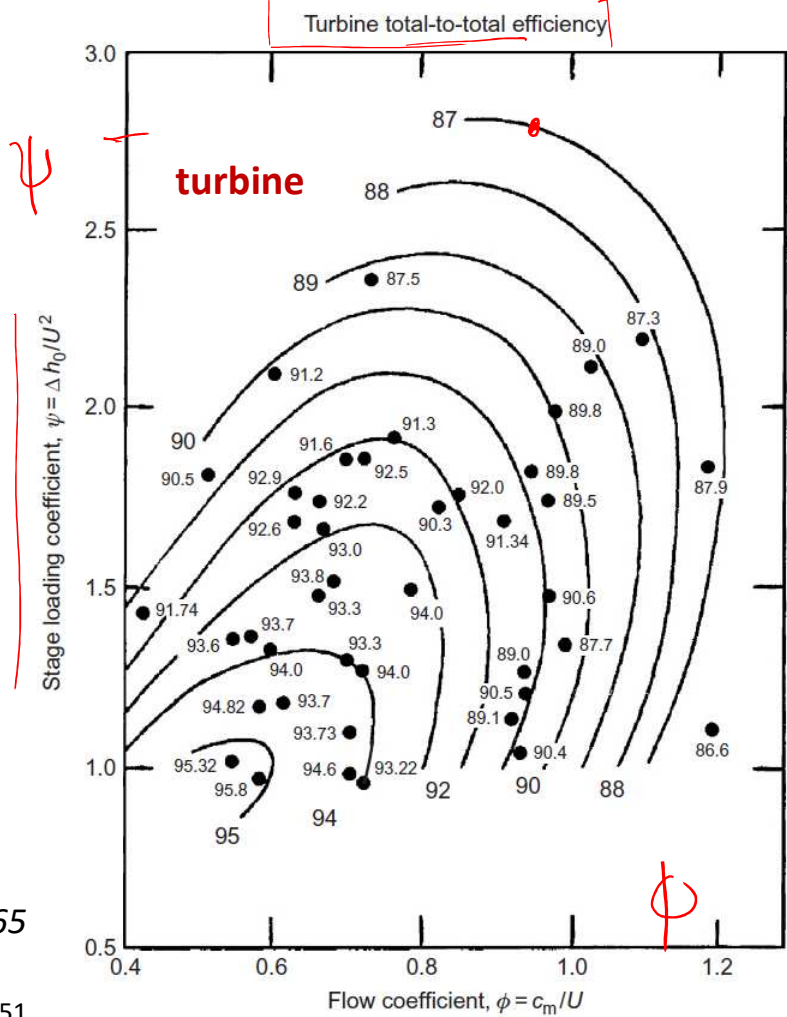
gas-specific constant, dry air in J/(kg.K)

(universal gas constant)
 $\bar{R} = 8.314 \text{ J / (mol}\cdot\text{K)}$

$$\Rightarrow \underline{\underline{\dot{m} = 4036 \text{ kg/s}}}$$

Turbomachinery

Interpretations using graphical data \times n_t



- you might be given a plot of this kind and be asked to interpret it, or use values from such a plot in your stage calculations

